SOME EFFECTS OF FORMATION ANISOTROPY ON RESISTIVITY MEASUREMENTS IN BOREHOLES*

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ABSTRACT

It is shown that a wide class of potential problems involving anisotropic media can be transformed into equivalent problems involving only isotropic media. By means of such transformations it is possible, in a large number of cases, to determine the apparent resistivities which would be observed in anisotropic formations, using electrode-type resistivity logging devices. Discussion is given of an infinite, anisotropic medium with and without borehole, of two semi-infinite anisotropic beds (without borehole), and of a thin isotropic bed bounded by anisotropic adjacent formations (without borehole). An interpretation chart for the normal device is presented for thick, noninvaded, anisotropic beds penetrated by a borehole.

INTRODUCTION

In many sedimentary strata, electric current flows more easily in a direction parallel to the bedding planes than transversely to them. The reason for this anisotropy is that a great number of mineral crystals possess a flat or elongated shape (e.g., mica and kaolin). At the time they were laid down, they naturally took an orientation parallel to the plane of sedimentation. The interstices in the formations are, therefore, parallel to the bedding plane, and the current is able to travel with greater facility along these interstices which generally contain mineralized water. Such electrical anisotropy is observed mostly in shales.

If a cylindrical sample is cut from a formation, parallel to the bedding planes, the resistivity of this sample measured with a current flowing along its axis is called the longitudinal resistivity R_H. If a similar cylinder is cut perpendicular to the bedding, the resistivity measured with a current flowing along its axis is called the transversal resistivity R_V. The anisotropy coefficient λ, by definition, is equal to $\sqrt{R_V/R_H}$. Laboratory measurements have shown that λ may range from 1 to about 2.5 in different shales.

Furthermore, the formations are often made up of a series of relatively thin beds having different lithologic characteristics and, therefore, different resistivities (as, for example, sequences of thin shales and hard streaks). In electrode systems used for electrical prospecting and for electrical logging, the distances between the electrodes are great enough that the volume involved in a measurement may include several such thin beds. Since, in this situation, the current flows more easily along the more conductive streaks than transversely to the series of beds, there is effective anisotropy. The effects on resistivity measurements of this "macroscopic" anisotropy are added to the effects of the anisotropy due to the microscopic structure of the sediments which was described above.

The electrical anisotropy of sedimentary formations was recognized by

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C. Schlumberger (1920) early in the history of electrical prospecting. Later, in 1932, a method of computing the electric field in anisotropic media was presented by R. Maillet and H. G. Doll. Practical use has also been made of the anisotropic properites of sediments for the determination of the directions of the strike and the dip, both in surface prospecting and in well logging (Schlumberger and Schlumberger, 1930; Schlumberger, Schlumberger, and Leonardon, 1934; Schlumberger, Schlumberger, and Doll, 1933).

One important result brought to light in the papers by Schlumberger, Schlumberger, and Leonardon (1934) and by Maillet and Doll (1932) is that the resistivity measured with a system of electrodes aligned perpendicularly to the bedding planes is equal to the longitudinal resistivity (paradox of anisotropy). In the particular case of electrical logging in boreholes, the measuring device is generally vertical, and, whenever the formation dip is small or nil, the measured resistivity accordingly is equal to the longitudinal resistivity, provided the electrode spacing is large enough for the effect of the borehole to be negligible.

Recently, some features observed on resistivity logs have called attention to the need for a more complete investigation of the effect of anisotropy. One of these phenomena is the difference in readings between the Induction Log[1] and the 16-inch Normal[2] at the level of certain shales (the former being systematically lower than the latter). Another phenomenon is the abnormally low values read with the lateral device[2] at the level of certain sands bounded by shales.

A mathematical study has accordingly been made of the effect of anisotropy on resistivity logs made with electrode devices, with special attention to the following cases:

a) Homogeneous, infinitely thick, anisotropic formations, traversed by a borehole;
b) Sequences of horizontal, homogeneous, anisotropic beds, borehole effect neglected.

It will be shown that the evaluation of the anisotropy effect is possible by substituting for each anisotropic formation an appropriate isotropic medium. Using such equivalences, the responses of normal and lateral devices in anisotropic media can often be determined.

Departure curves, giving the apparent resistivity read by a normal device in a thick anisotropic bed as a function of the electrode spacing, are presented for several values of the anisotropy coefficient. These curves take into account

[1] As explained in detail in Doll (1949), in measurements of resistivity by Induction Logging, current loops encircling the axis of the hole are induced in the earth. Since, for a vertical drill hole, in the absence of formation dip, these currents circulate horizontally, their flow is affected only by the longitudinal resistivity; and the reading so found is a measure of R_H.

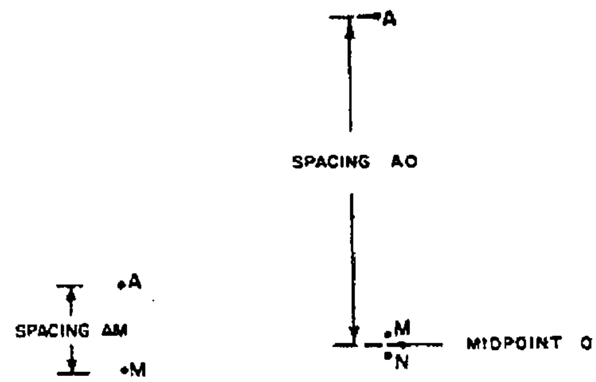
[2] For the arrangement of the current and measuring electrodes in normal and lateral resistivity logging devices, see Figure 1. For a more detailed explanation, see Chapter I, Section 2, of the Schlumberger Corporation's *Interpretation Hand-Book for Resistivity Logs*. A 16-inch Normal is a normal device with an AM spacing of 16 inches.

the presence of a borehole and have been numerically computed using an IBM 701 Data Processing Machine.

The behavior of the normal and lateral curves is discussed for the case of two semi-infinite media with a common boundary, and for the case of a thin bed.

GENERAL EQUATIONS FOR CYLINDRICAL SYMMETRY

On the assumption that all the beds are horizontal and the borehole is vertical, one may speak of a horizontal (longitudinal) resistivity R_H parallel to the bedding



(a) NORMAL (b) LATERAL

FIG. 1. Electrode designations for the simplified normal and lateral devices referred to in this paper. A designates a current electrode, whereas M and N designate measure electrodes. A current return electrode, B, is located at a great distance from the hole device proper. In the idealized lateral, assumed in the computations, measure electrodes M and N are very close together.

To simplify the discussion, the electrodes are treated as points. No consideration is made of the size of the electrodes or the presence of the insulating mandrel on which they are mounted. The effects of these, however, are minor.

planes and of a vertical (transversal) resistivity R_V perpendicular to these planes. If the current sources lie on the axis of the borehole, the potential surfaces and the lines of current flow possess a cylindrical symmetry.

In a cylindrical co-ordinate system (ρ, z, θ), Ohm's law may be generalized to state that the current density J is given in terms of the potential V by the equation

$$-J = \left[\frac{1}{R_H}\frac{\partial V}{\partial \rho}\rho_1 + \frac{1}{R_V}\frac{\partial V}{\partial z}z_1\right] \quad (1)$$

where z_1 is a unit vector parallel to the axis of the borehole (z-axis) and ρ_1 a unit vector perpendicular to z_1 and directed away from the borehole axis.

The normals to the equipotential surfaces, $V(\rho, z) =$ constant, are parallel to the gradient of V, given by

$$\nabla V = \frac{\partial V}{\partial \rho}\rho_1 + \frac{\partial V}{\partial z} z_1. \tag{2}$$

Thus, in general, the current flow in an anisotropic formation ($R_V \neq R_H$) will not be normal to the potential surfaces.

The absence of current sources or sinks, everywhere except on the borehole axis, is expressed by requiring that the divergence of the current density vanishes for $\rho \neq 0$. This leads to the differential equation

$$\nabla \cdot J = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\frac{\rho}{R_H}\frac{\partial V}{\partial \rho}\right) + \frac{\partial}{\partial z}\left(\frac{1}{R_V}\frac{\partial V}{\partial z}\right) = 0. \tag{3}$$

For a homogeneous, isotropic medium, in which $R_H = R_V =$ constant, this equation reduces to the Laplace equation in cylindrical co-ordinates.

It is convenient to introduce the *mean* resistivity R and the *coefficient of anisotropy* λ, defined by setting

$$\begin{aligned} R &= \sqrt{R_H \cdot R_V} \\ \lambda &= \sqrt{R_V / R_H} \end{aligned} \tag{4}$$

Then

$$R_H = \frac{R}{\lambda}, \qquad R_V = \lambda R. \tag{5}$$

If it is assumed that the variation of the coefficient of anisotropy with ρ and z is expressible in the form

$$\lambda = f(\rho) \cdot g(z), \tag{6}$$

then, by (3)

$$\frac{f}{\rho}\frac{\partial}{\partial \rho}\left(\frac{f\rho}{R}\frac{\partial V}{\partial \rho}\right) + \frac{1}{g}\frac{\partial}{\partial z}\left(\frac{1}{gR}\frac{\partial V}{\partial z}\right) = 0. \tag{7}$$

By introducing the new variables

$$\rho' = \int_0^\rho \frac{d\rho}{f(\rho)} \quad \text{and} \quad z' = \int_0^z g(z)dz \tag{8}$$

and letting

$$V(\rho, z) = V'(\rho', z'), \tag{9}$$

one finds that

$$\frac{\partial V}{\partial \rho} = \frac{1}{f}\,\frac{\partial V'}{\partial \rho'} \tag{10}$$

and

$$\frac{\partial V}{\partial z} = g\,\frac{\partial V'}{\partial z'}. \tag{11}$$

With these transformations, (7) becomes

$$\frac{\partial}{\partial \rho'}\left(\frac{\rho}{R}\,\frac{\partial V'}{\partial \rho'}\right) + \frac{\partial}{\partial z'}\left(\frac{\rho}{R}\,\frac{\partial V'}{\partial z'}\right) = 0. \tag{12}$$

Finally, by introducing a new value of resistivity at each point, given by

$$R' = \frac{\rho'}{\rho}\,R, \tag{13}$$

one may replace (12) by

$$\frac{1}{\rho'}\,\frac{\partial}{\partial \rho'}\left(\frac{\rho'}{R'}\,\frac{\partial V'}{\partial \rho'}\right) + \frac{\partial}{\partial z'}\left(\frac{1}{R'}\,\frac{\partial V'}{\partial z'}\right) = 0. \tag{14}$$

A comparison of equation (14) with (3) shows that in terms of the primed co-ordinates (ρ', z') the differential equation for the potential V' is that required for an isotropic medium of resistivity R'. Therefore, *within the conditions imposed by* (6), *any medium involving anisotropic formations may be replaced by an equivalent isotropic one*. This is done by mapping each point (ρ, z) in the original space into a corresponding point (ρ', z') of a new space. In this mapping the potential remains the same at corresponding points (see equation (9)), but the mean resistivity changes from R to R' $(=\rho' R/\rho)$. It remains necessary only to investigate how the boundary conditions are to be transformed.

TRANSFORMATION OF BOUNDARY CONDITIONS

The boundary conditions of importance here are those involving the continuity of potential and of the normal component of current across various boundaries. The continuity of potential in the primed system is insured immediately by (9), and thus we need only to consider the transformation of the current across equivalent sections of a given boundary.

The current flowing out through the sides of any cylinder of radius ρ whose axis is that of the borehole, is given by

$$I_H = -2\pi\rho \int_{z_1}^{z_2} \frac{1}{R_H}\,\frac{\partial V}{\partial \rho}\,dz, \tag{15a}$$

where z_1 and z_2 are the z co-ordinates of top and bottom of the cylinder. The current I' flowing out of the corresponding cylinder of radius ρ' in the transformed space is given by

$$I'_H = -2\pi\rho' \int_{z_1'}^{z_2'} \frac{1}{R'} \frac{\partial V'}{\partial \rho'} dz'. \tag{15b}$$

Direct substitution from equations (5), (6), (8), (10), and (13) proves that $I_H = I'_H$.

The current flowing in the positive z-direction through any plane horizontal annular surface, bounded by circles of radii ρ_1 and ρ_2, is given by

$$I_V = -2\pi \int_{\rho_1}^{\rho_2} \frac{\rho}{R_V} \frac{\partial V}{\partial z} d\rho \tag{16}$$

which again can be shown to be equal to the current I'_V through the corresponding annular surface in the primed system. Using these results, it is easy to show that the transformation described by equations (8), (9), and (13) leaves unchanged the current flowing through corresponding surfaces. In particular, this means that the strength of any current source or sink will not be changed.

AN INFINITE, HOMOGENEOUS, ANISOTROPIC MEDIUM

For such a medium, λ = constant, and one can, by (6), make one of the following choices:

$$f(\rho) = 1, \qquad g(z) = \lambda \tag{17a}$$

or

$$f(\rho) = \lambda, \qquad g(z) = 1. \tag{17b}$$

By equations (8) and (13) the first choice leads to an expansion in the z-direction given by $z' = \lambda z$ and a resistivity R' of the equivalent medium equal to the mean resistivity R; whereas the second choice corresponds to a radial compression given by $\rho' = \rho/\lambda$ and an equivalent homogeneous resistivity given by $R' = R/\lambda = R_H$.

The transformation to an isotropic medium represented by the second choice, since it involves no change in the electrode spacings, shows at once that theoretically, *if the borehole could be neglected*, a normal or lateral device would measure R_H, the horizontal resistivity (paradox of anisotropy).

For the first choice, which corresponds to a transformation to an isotropic medium having a resistivity equal to the mean resistivity R, it is necessary only to take into account the increase by a factor λ in the equivalent electrode spacing. This increase in the spacing will mean that the potential measured by a normal sonde, and hence the apparent resistivity measured by it, will be reduced by the factor $1/\lambda$, and thus one again measures $R_H = R/\lambda$. For the lateral sonde

the increase in the spacing AO reduced the signal by $1/\lambda^2$, but the increase in the measure electrode span MN increases the potential difference between them by λ, and the horizontal resistivity R_H is again measured.

EFFECT OF THE BOREHOLE

Subsurface resistivity measurements with an electrode device are generally made by lowering the device into a drilled hole filled with drilling mud. The presence of the electrically conductive (and isotropic) column of drilling mud containing the device can be taken into account in equation (6) by noting that

$$\lambda = \begin{cases} 1 & \text{for} \quad \rho < a \\ \lambda_0 & \text{for} \quad \rho > a, \end{cases} \tag{18}$$

where a is the radius of the borehole. Also, since λ is a function of ρ, one sets $f(\rho) = \lambda$ and $g(z) = 1$; thus, from (8),

$$z' = z \tag{19a}$$

and

$$\rho' = \left\{ \begin{matrix} \rho & \text{for } \rho < a \\ a + (\rho - a)/\lambda_0 & \text{for } \rho > a \end{matrix} \right\}. \tag{19b}$$

The equivalent isotropic resistivity after the transformation is, from equation (13),

$$R' = \begin{cases} R_m & \text{for } \rho < a \\ \dfrac{R\rho'}{\lambda_0\rho' - (\lambda_0 - 1)a} & \text{for } \rho > a, \end{cases} \tag{20}$$

where R_m is the resistivity of the mud filling the hole, and R is the mean resistivity of the anisotropic formation. The radial resistivity profile as given by (20) is shown in Figure 2. Note that, as $\rho' \to \infty$, the resistivity R' of the equivalent isotropic formation approaches asymptotically $R_H = R/\lambda_0$, the horizontal resistivity. At the edge of the borehole ($\rho' = a$), as seen from (20), R' is equal to R; but at

$$\rho' = \left(1 + \frac{1}{\lambda_0}\right)a,$$

which is less than twice the borehole radius, $R' = \frac{1}{2}(R + R_H)$. At this distance R' has returned halfway to its asymptotic value R_H.

From this analysis the departure curves for normal and lateral devices should lie above the isotropic curves for a formation of resistivity R_H and below those for a formation of resistivity $R = \lambda_0 R_H$, and they should approach the former for large spacings. This problem is treated quantitatively in the next section, and, as seen in Figure 3, these characteristic features are indeed borne out. One is led,

therefore, to believe that the transformation to an equivalent isotropic formation will often serve to gain a qualitative prediction of the apparent resistivity to be expected.

QUANTITATIVE STUDY OF THE BOREHOLE

It has been shown above that a normal or lateral sonde in an infinite, homogeneous, anisotropic medium will measure R_H and that the effect of the borehole through such a medium is to increase the observed apparent resistivity

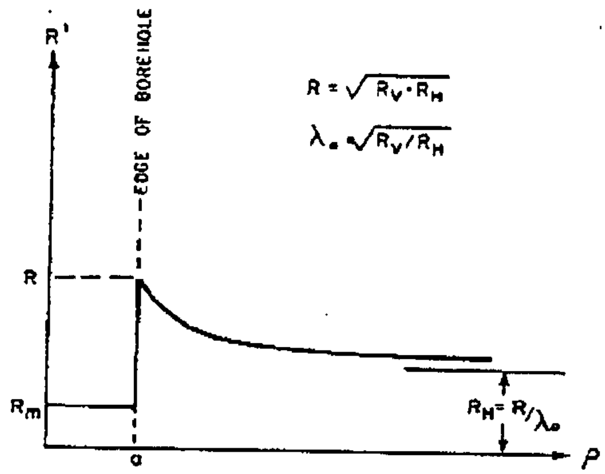


FIG. 2. Radial plot of the resistivity R' of an isotropic medium equivalent to a homogeneous anisotropic medium pierced by an isotropic homogeneous mud column of resistivity R_m. (Plotted for $\lambda_e = 2$.)

as compared to a homogeneous, isotropic formation of resistivity R_H. In fact, the formation will appear as an isotropic medium with a resistivity lying between R_H and λR_H, depending on the spacing.

This problem will now be formulated quantitatively and some numerical results given. Let the potential in the borehole be V_m while that in the formation is V_f. Then a solution of (3) must be found subject to the boundary conditions:

$$\text{(a)} \qquad V_m \to \frac{IR_m}{4\pi r} \qquad \text{as } r \to 0,$$

$$\text{(b)} \qquad V_m, V_f \to 0 \qquad \text{as } r \to \infty, \qquad (21)$$

$$\text{(c)} \qquad \frac{1}{R_m}\frac{\partial V_m}{\partial \rho} = \frac{1}{R_H}\frac{\partial V_f}{\partial \rho} \qquad \text{at } \rho = a,$$

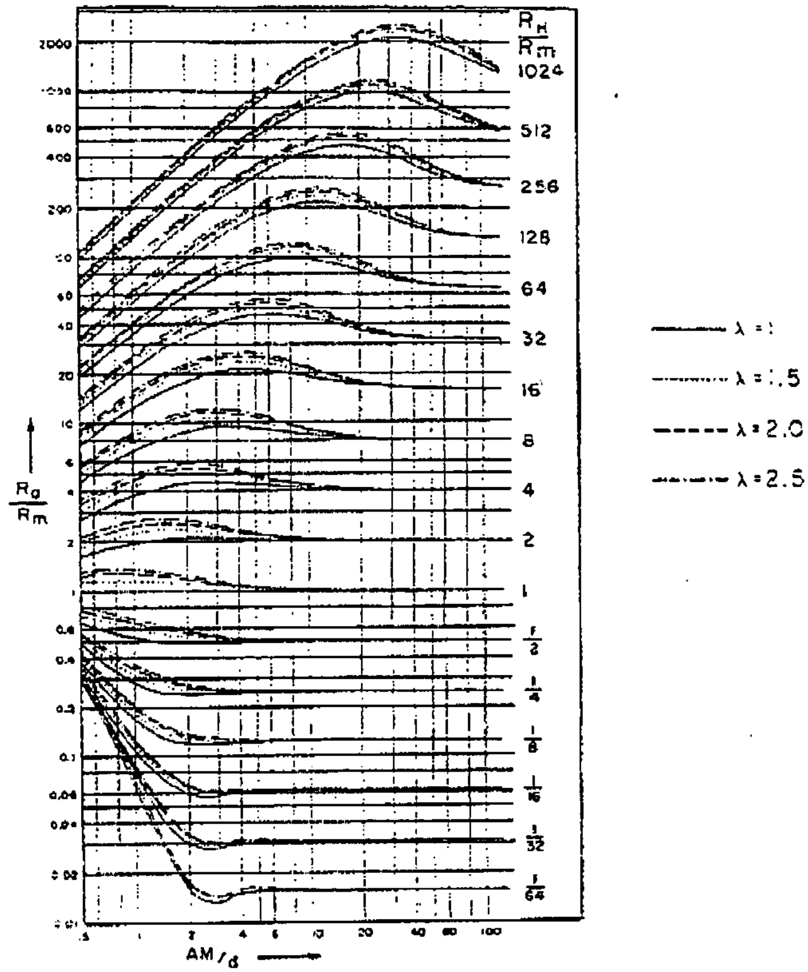


FIG. 3. Normal device (no invasion).

resulting formula has the same form as Simpson's formula except that the weights to be assigned to the ordinates depend on the magnitude of zh/a and reduce to Simpson's weights only when zh/a is small compared to one, h being the width of the z-intervals in the numerical integration.

This method of evaluating the integral in (24) was programmed for the IBM 701 Data Processing Machine and was found to give satisfactory results even for a highly oscillatory integrand corresponding to $z/a = 256$.

The results of these computations are shown in Figure 3 as a set of departure curves for the normal device. Curves are plotted for various values of R_H/R_m (ratio of horizontal formation resistivity to mud resistivity) for four degrees of anisotropy ($\lambda = 1, 1.5, 2,$ and 2.5). Each curve shows the variation of R_a/R_m (ratio of apparent resistivity to mud resistivity) versus AM/d (ratio of normal spacing to hole diameter).

The comments on an infinite, homogeneous, anisotropic medium are seen to describe the qualitative features of the curves quite accurately. For a given R_H/R_m, increased anisotropy (increased λ) corresponds in general to an increased R_a/R_m. (This explains why the 16-inch Normal reading at the level of shales may be higher than the Induction Log reading, as mentioned in the Introduction.) For spacings large in comparison with the hole diameter (AM/d large), the apparent resistivity R_a approaches R_H.

It was possible to use this same approach to compute the effect of anisotropy when the formation has been invaded with mud filtrate. The formation close to the borehole will have a different resistivity from the uncontaminated part of the formation further back from the borehole. The formulas are then, of course, more complicated, particularly if the coefficient of anisotropy in the invaded zone is not assumed to be the same as in the uncontaminated zone.

GENERAL CASE NEGLECTING THE BOREHOLE AND INVASION

If the influence of the borehole is neglected, the transformation to an isotropic medium can be made by setting

$$f(\rho) = 1 \quad \text{and} \quad g(z) = \lambda \tag{25}$$

in equation (6). The transformation of equations (8) and (13) then involves only a stretching in the z-direction and no change in the radial distance or the mean resistivity. Each anisotropic bed will be replaced by an isotropic bed having the same mean resistivity, but the thickness of the bed will be multiplied by the anisotropy coefficient λ. Also, of course, after such a transformation, one must take for the new spacings between the electrodes of a sonde the differences in their z'-coordinates, as given by (8).

The above transformation makes it possible to reduce the problem of finding the potentials, or potential differences, to be observed on the measuring electrodes in the anisotropic case to that of finding the corresponding potentials in an equivalent isotropic case. To express the results in terms of the apparent resistivity R_a recorded on a log, however, use is made of the linear relation

$$R_a = KV/I \tag{26}$$

(d) $$V_m = V_f \qquad \text{at } \rho = a,$$

where

$$r = \sqrt{\rho^2 + z^2} \text{ and } a \text{ is the borehole radius.}$$

Condition (a) specifies the source as an isotropic current emitter located at the center of the borehole. Condition (b) simply fixes the potential at infinity. Condition (c) is the statement of continuity of the normal component of current density across the cylinder $\rho = a$, representing the wall of the borehole. Condition (d) is a statement of the continuity of potential across the cylinder $\rho = a$ and is equivalent to the statement that there is no charge double layer on this surface.

A formal solution of this problem can be constructed in a fashion entirely analogous to the treatment given in the Appendix of the Schlumberger Corporation's *Interpretation Hand-Book for Resistivity Logs*. The notation adopted is that in the previous section, but now the origin is at the current electrode, and thus z is the spacing of the normal device. The potential in the borehole, expressed in terms of the modified Bessel functions K_0, K_1, I_0, and I_1, is found to be

$$V_m(\rho, z) = \frac{IR_m}{4\pi}\left[\frac{1}{r} + \frac{2}{\pi}\int_0^\infty A(x) I_0(x\rho/a) \cos(xz/a)\frac{dx}{a}\right], \tag{22}$$

where

$$A(x) = \frac{K_0(x/\lambda)K_1(x) - \dfrac{R_m}{\lambda R}K_0(x)K_1(x/\lambda)}{K_0(x/\lambda)I_1(x) + \dfrac{R_m}{\lambda R}I_0(x)K_1(x/\lambda)}.$$

The "apparent" resistivity for the normal device is given by the formula[3]

$$R_a = \frac{4\pi z}{I} V(0, z). \tag{23}$$

The essential step in the computation of (23) is the evaluation of the Fourier cosine integral which appears in (23) when $V(0, z)$ is inserted from (22), namely:

$$F\left(\lambda, \frac{R}{R_m}, \frac{z}{a}\right) = \frac{2}{\pi}\int_0^\infty A(x)\cos(xz/a)\,dx. \tag{24}$$

An effective technique for the numerical evaluation of such an integral for large values of z/a and a slowly varying function $A(x)$ has been described by L. N. G. Filon (1928–1929).

Filon's suggestion amounts to fitting $A(x)$ with a set of parabolic arcs. The

[3] The factor $4\pi z$ is the K of the normal device. See equation (20) and associated discussion.

between R_a and the measured[4] potential difference, V. The proportionality constant K is so chosen that R_a is equal to the true resistivity for an infinite, homogeneous, isotropic medium. For the normal sonde, K is proportional to the spacing AM, whereas for the lateral it is proportional to $(AM)\cdot(AN)/(MN)$. Thus, if one computes directly the potential difference to be expected in any given case, by solving the equivalent isotropic problem above, this potential difference may be substituted for V in equation (25) to obtain R_a.

In some cases the equivalent isotropic problem may have already been solved, as on existing interpretation charts (from resistor network measurements, experiments, or computations), giving the effect of electrode spacing, bed thickness, etc. Such data will ordinarily be presented in terms of an apparent resistivity

$$R_a' = K'V/I, \tag{27}$$

where the coefficient K' depends on the spacings used *in the equivalent isotropic problem*.

It is necessary, in using such data, to know whether the K' of the equivalent problem is the same as the K in the original problem or whether a transformation of the K is involved. In the present case (in which the z-co-ordinates, and hence the electrode spacings, are transformed) there has indeed been a transformation of the K.

The apparent resistivity data of the equivalent case can be used to obtain the potential difference V needed in (22) if care is taken that at each depth the K' must correspond to the transformed spacings. Substituting this V into (22), it is seen that the apparent resistivity R_a (for the given anisotropic problem) is related to the apparent resistivity R_a' (for the equivalent isotropic problem) through the equation

$$R_a = \frac{K}{K'} R_a'. \tag{28}$$

Note that K is determined by the actual sonde spacings in the original problem, whereas K' is determined by the transformed spacings in terms of the z'-co-ordinates.

In particular, for the normal sonde, in the case being discussed of no borehole and no invasion,

$$\frac{K}{K'} = \frac{z_M - z_A}{\int_{z_A}^{z_M} g(z)dz} = \frac{1}{\bar{\lambda}}, \tag{29}$$

where $\bar{\lambda}$ is the average of λ between the current electrode A at $z=z_A$ and the measure electrode M at $z=z_M$.

[4] For the normal sonde the potential difference referred to is that between the electrode M and a reference electrode N at the surface. See the Schlumberger Corporation's *Interpretation Hand-Book for Resistivity Logs*, p. 8-11.

For a lateral sonde having a short MN span, the K' coefficient is very nearly proportional to $(AO)^2/MN$, as measured in the equivalent isotropic medium, and, therefore,

$$\frac{K}{K'} = \left[\frac{z_O - z_A}{\int_{z_A}^{z_O} g(z)\,dz}\right]^2 g(z_O) = \frac{\lambda}{\bar{\lambda}^2}, \tag{30}$$

where $\lambda = g(z_O)$ is the value of the anisotropy coefficient at O, and $\bar{\lambda}$ is its average value between A and O.

QUANTITATIVE STUDY OF THE TWO-MEDIA PROBLEM WITHOUT BOREHOLE

The problem considered here belongs to the class treated in the above section. It is assumed that two thick anisotropic beds have a common boundary, and it is desired to evaluate the response of the normal and lateral devices near this boundary. In the first bed the horizontal and vertical resistivities are R_{H_1} and R_{V_1}, respectively. The mean resistivity, $R_1(=\sqrt{R_{V_1}\cdot R_{H_1}})$ and the anisotropy coefficient $\lambda_1(=\sqrt{R_{V_1}/R_{H_1}})$ are defined in agreement with equation (4). With similar definitions for the second bed, the original problem can be replaced by an equivalent isotropic problem, in the manner indicated in the previous section (equation (25)).

The current electrode A and a point of measurement M are assumed to be located on the same normal to the boundary between the two media. The upper half of Figure 4 shows three different versions of the problem: (a) Electrodes A and M are in medium no. 1; (b) electrode A is in medium no. 1, and electrode M is in medium no. 2; or (c) both electrodes are in medium no. 2. (In each case shown, A is above M. However, the same end result is found for the potential if A and M are simply interchanged in position.) The equivalent isotropic cases are pictured in the lower half of Figure 4.

Using L as the distance from electrode A to the boundary and z as the vertical co-ordinate of M with reference to A (positive downward), the potential in the two beds can be found from the equivalent isotropic cases, by the method of images.

$$\left.\begin{aligned}
V &= \frac{IR_1}{4\pi\lambda_1}\left[\frac{1}{|z|} + \frac{R_2 - R_1}{R_2 + R_1}\left(\frac{1}{|2L - z|}\right)\right] && \text{for } A \text{ and } M \text{ in medium no. 1}\\
V &= \frac{I}{4\pi}\cdot\frac{2R_1R_2}{R_1 + R_2}\,\frac{1}{|\lambda_1 L + \lambda_2(z - L)|} && \text{for } A \text{ in medium no. 1, } M \text{ in medium no. 2}\\
V &= \frac{IR_2}{4\pi\lambda_2}\left[\frac{1}{|z|} + \frac{R_1 - R_2}{R_1 + R_2}\left(\frac{1}{|2L + z|}\right)\right] && \text{for } A \text{ and } M \text{ in medium no. 2}
\end{aligned}\right\} \tag{31}$$

Using these relations, the apparent resistivities which would be given by the

normal and lateral devices may be found.[3] For this determination, the coordinate z is set equal to the spacing of the appropriate normal device.

Special Case $R_2 = R_1$

This is a case of special interest because the equivalent isotropic problem corresponds to a homogeneous medium. The apparent resistivity profiles versus depth for such a case are illustrated by the solid curves, in Figure 5 for the normal device, and in Figure 6 for the lateral. In each figure the upper anisotropic bed

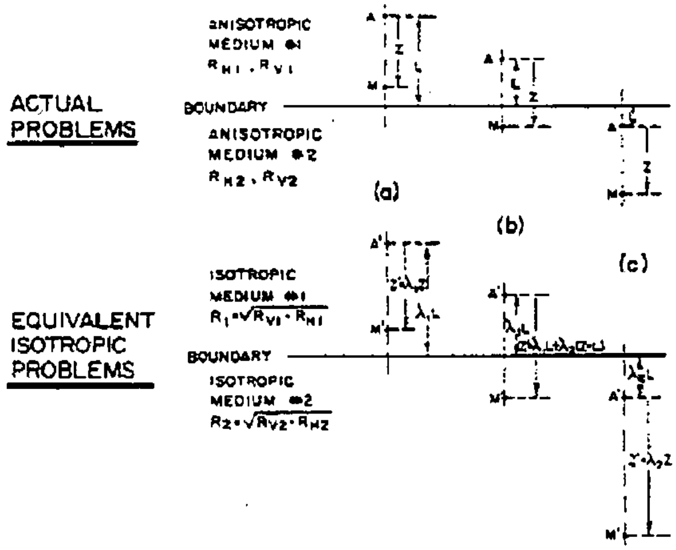


FIG. 4. Two semi-infinite anisotropic media, borehole effect neglected. (Drawn for $\lambda_1 = 1.25$, $\lambda_2 = 2$.)

(region 1) has a horizontal resistivity R_{H_1} of 2, and the lower anisotropic bed (region 2) has R_{H_2} equal to 1. (Note that the reference points for depths have been shifted to the midpoint of AM for the normal, and to point O for the lateral.)

[3] The normal apparent resistivity is computed from equation (23) (or equivalently equation (26)). In the present and following sections, the lateral apparent resistivity is computed from the relation

$$(R_a)\ \text{lateral} = \frac{-4\pi s^2}{I} \frac{\partial V}{\partial s}$$

where s is set equal to the spacing AO. This relation is a good approximation when MN is negligible in comparison with AO.

Also shown for comparison in Figures 5 and 6, in dashed line, are the apparent resistivity profiles for two isotropic beds whose resistivities R_1^* (upper bed) and R_2^* (lower bed) are equal to the *horizontal* resistivities of the corresponding anisotropic beds; i.e., $R_1^* = R_{H_1}$ and $R_2^* = R_{H_2}$. (R_1^* and R_2^* are not equivalent isotropic resistivities.)

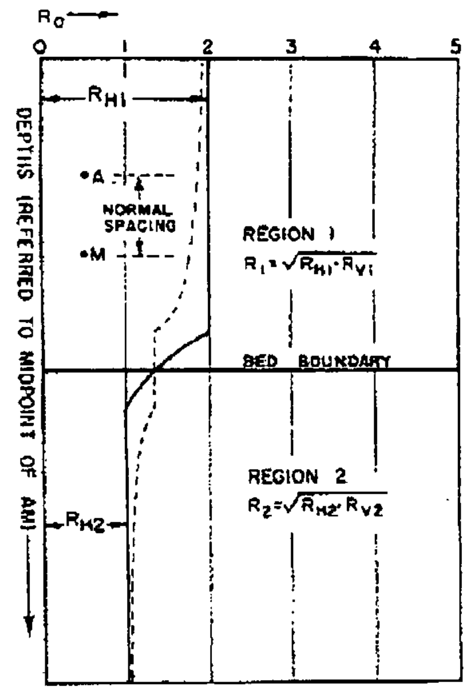


FIG. 5. Normal sonde, borehole effect neglected.

Both normal and lateral curves for the anisotropic case show values of R_a which are constant and equal to $R_{H_1} (= R_1/\lambda_1)$, until the measure electrodes cross the boundary. Then, for the normal, while the A electrode is in one medium and the measure electrode in the other, the value of the potential makes a smooth transition from $IR_{H_1}/4\pi s$ to $IR_{H_2}/4\pi s$. This transition from an apparent resistivity R_{H_1} to one of R_{H_2} is completed in the distance $s(= AM)$, which is one sonde spacing.

For the lateral, when the measure electrodes cross the bed boundary, the apparent resistivity makes a jump such that the readings on the two sides of the boundary are in the ratio λ_1/λ_2 (equal in this case to R_{H_2}/R_{H_1}). This sudden

jump is followed by a smooth transition to the value $R_{H_2}(=R_2/\lambda_2)$, which takes place in the distance of one sonde spacing AO.

This example illustrates some of the novelty due to anisotropy. If we attempted to interpret the apparent resistivity curve in this case as though the media were isotropic, we would have to assign resistivities R_{H_1} and R_{H_2} to match

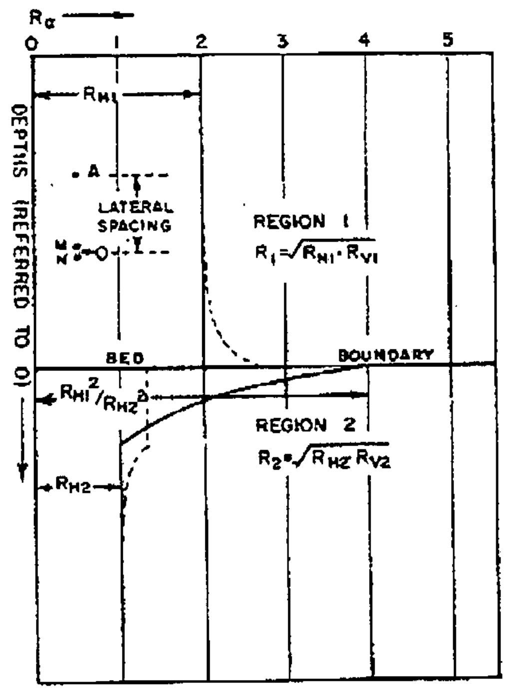


FIG. 6. Lateral sonde, borehole effect neglected.

the curve at a distance from the boundary. Then the behavior at the boundary would be misleading. (Compare dashed and solid curves in Figures 5 and 6.)

Special Case $R_{H_1}=R_{H_2}$, *but* $R_1 \neq R_2$

The apparent resistivity R_a will be the same in both media except in the immediate neighborhood of the boundary. Let us suppose that $\lambda_2>\lambda_1$. Then, as indicated in Figure 7, the reading of the normal device will increase as the boundary is approached from above, reaching a value

$$\frac{2\lambda_2}{\lambda_1+\lambda_2}R_H$$

when the measure electrode arrives at the boundary. After the measure electrode crosses the boundary, the apparent resistivity decreases smoothly, becoming equal to $2\lambda_1 R_H/(\lambda_1+\lambda_2)$ when the current electrode reaches the boundary. The reading then increases to the value R_H as the normal device continues to move downward.

The lateral device under the same circumstances will show a decreasing read-

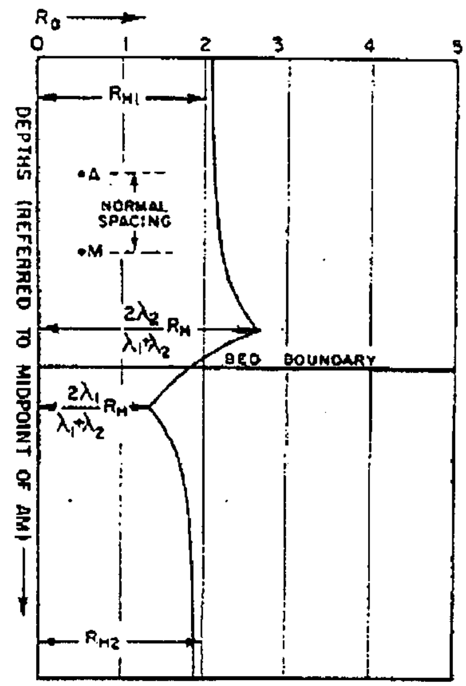


FIG. 7. Normal sonde, borehole effect neglected. Special case of $R_{H_1}=R_{H_2}=R_H$. (Plotted for $\lambda_1=1$, $\lambda_2=2$, $R_H=2$.)

ing, as shown in Figure 8, arriving at a value $2\lambda_1 R_H/(\lambda_1+\lambda_2)$ when the measure electrodes are just above the boundary. When the measure electrodes cross the boundary, the apparent resistivity jumps up by a factor of $(\lambda_2/\lambda_1)^2$. It then decreases, reaching a second minimum when the current electrode passes the boundary. (The minima have the same value as the one for the normal device.) Thereafter the apparent resistivity increases toward its final value of R_H.

For both of the special cases illustrated in Figures 5 to 8 inclusive, it has been

assumed that $\lambda_2 > \lambda_1$. In a case where $\lambda_1 < \lambda_2$, the character of the features would be reversed. For example, in Figure 8 the dip in the lateral just above the upper bed boundary would become a peak. The peak just below the upper bed boundary would become a dip. The dip at one sonde spacing below the bed boundary would become a peak.

The resistivity profiles of Figures 5 to 8 furthermore indicate that the response

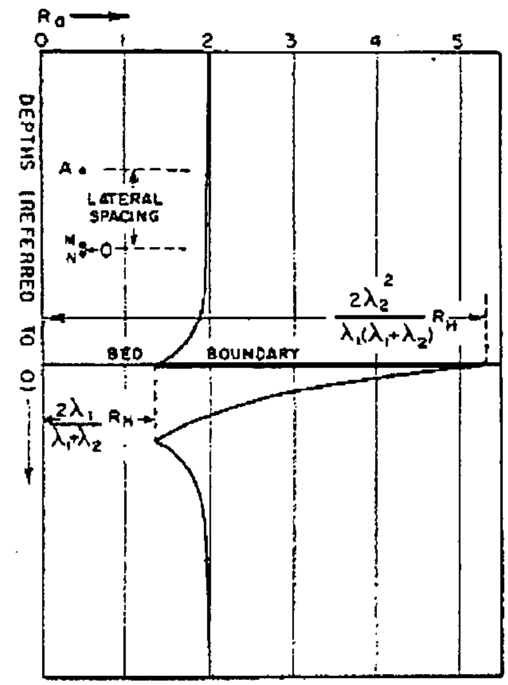


FIG. 8. Lateral sonde, borehole effect neglected. Special case of R_{H_1} R_{H_2} R_H.
(plotted for $\lambda_1 = 1$, $\lambda_2 = 2$, $R_H = 2$.)

of the lateral device is more strikingly affected by the presence of anisotropy than the response of the normal.

General Case

The most interesting features on the apparent resistivity curve are those observed when the measure electrodes are near the boundary and when the current electrode penetrates the boundary. These values are listed below in terms of the

"mean" resistivity $\bar{R}$, defined by

$$\frac{1}{\bar{R}} = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

$$\left.\begin{array}{lll} R_a = \dfrac{\bar{R}}{\lambda_1} & \text{measure electrode at boundary} & \\ \quad = \dfrac{2\bar{R}}{\lambda_1 + \lambda_2} & \text{midpoint of } AM \text{ at boundary} & \text{NORMAL DEVICE} \\ \quad = \dfrac{\bar{R}}{\lambda_2} & \text{current electrode at boundary} & \\ R_a = \dfrac{\bar{R}}{\lambda_1}\left(\dfrac{R_1}{R_2}\right) & \text{measure electrodes just above boundary} & \\ \quad = \dfrac{\bar{R}}{\lambda_1}\left(\dfrac{\lambda_2}{\lambda_1}\right) & \text{measure electrodes just below boundary} & \text{LATERAL DEVICE} \\ \quad = \dfrac{\bar{R}}{\lambda_2} & \text{current electrode at boundary} & \end{array}\right. \qquad (32)$$

With these key values, a response curve may be sketched for any case of interest. If medium 2 is isotropic and we know R_{H_1} and R_2 from the readings above and below the boundary, then it is clear that readings corresponding to the lateral device response would allow the determination of $R_{V_1} (= \lambda_1 R_1)$ from the discontinuity in R_a at the boundary.

The fact that the lateral device shows a discontinuity upon crossing a boundary in the ratio R_{V_2}/R_{V_1} is quite a general result. It is, of course, dependent on the assumed smallness of the MN-span between the measuring electrodes and the absence of borehole and invasion.

THIN ISOTROPIC BED BETWEEN ANISOTROPIC ADJACENT FORMATIONS (SAND BED BOUNDED BY SHALES)

Using the technique of reducing the anisotropic problem to an equivalent isotropic problem, apparent resistivity profiles can be computed for the normal and lateral devices passing through isotropic beds, such as sands, bounded by anisotropic adjacent formations, such as shales. From these results, interpretation charts can be prepared. Only the lateral device will be considered here, because it seems clear that this device will be influenced more significantly by anisotropy.

That anisotropy in the adjacent formations may cause some ambiguity in determining the resistivity of a thin bed is apparent from the fact that at either boundary the lateral reading R_{a_B}, in the bed *just inside* the boundary, is related

to the reading R_{aS}, in the adjacent formation *just outside* the same boundary, by the relation

$$\frac{R_{aB}}{R_{aS}} = \frac{R_B}{R_V} = \frac{R_B}{\lambda^2 R_H} = \beta, \tag{33}$$

where R_V and R_H are the vertical and horizontal resistivities in the adjacent formations, λ is the coefficient of anisotropy in the adjacent formations, and R_B is the resistivity of the isotropic bed. At points well above or below the bed the apparent resistivity is $R_a = R_H$. If $R_B < \lambda^2 R_H$, equation (33) shows that R_a will decrease upon entering the bed, and this can occur even though $R_B > R_H$. Thus it is quite conceivable that a drop in R_a will correlate with an increase in actual (horizontal) resistivity. From (33) this can be expected to happen for

$$1/\lambda^2 < \beta < 1.$$

One might hope that this difficulty could be overcome by using other information from the resistivity curve, along with interpretation charts, to arrive at a better estimate of R_B than the one deduced directly from the log. This possibility will be considered.

Special Case $\lambda R_H = R_B$

Before considering the general case, however, let us take a look at a special problem that can be solved very simply by using the scaling technique of the sections which covered the general case (neglecting the borehole and invasion) and the quantitative study of the two-media problem without borehole. This special problem corresponds to the assumption that the mean resistivity of the anisotropic adjacent formation $R_s (= \sqrt{R_V \cdot R_H} = \lambda R_H)$ is equal to the bed resistivity R_B. The scaled problem then leads to a homogeneous medium of resistivity R_B, and, therefore, it can be solved easily. This approach leads to an apparent resistivity profile for the lateral sonde as sketched in Figure 9. In this figure, e is taken equal to the lateral spacing AO.

It is interesting to note that this curve could have been drawn directly from (28) and (30), since in this case $R_a' = R_B = R_S$ and the character of the curve is due completely to the variation of K'. The curve obtained in this case is typical of the result when $\beta < 1$, which is the case most likely to cause difficulty in log interpretation. If $\beta > 1$ the discontinuities are executed in the opposite directions.

General Case $\lambda R_H \neq R_B$

In generalizing to the case $R_S \neq R_B$, it is important to decide what data from the R_a curve should be used to determine R_B. It seems that for thin beds the best measurements would be:

(1) a measurement at the upper or lower boundary of the ratio β defined in (33);

(2) a measurement of R_a in the plateau when the sonde spans the bed. This value will be given the symbol R_P;

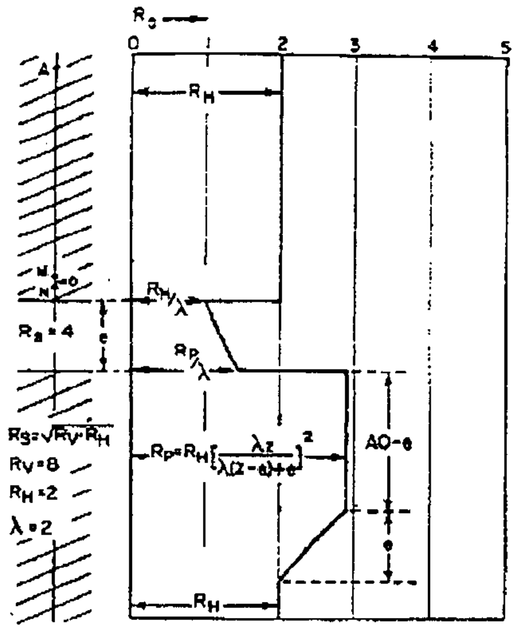


Fig. 9. Lateral sonde, borehole effect neglected. Thin isotropic bed between anisotropic shoulder formations for special case of $R_B = R_S$. ($R_B = R_S = 4$, $\lambda = 2$.)

(3) a measurement of R_H in the adjacent formations;

(4) a measurement of e, the bed thickness.

By (33), $R_B/R_H = \lambda^2\beta$, where β is known from measurement (1) above. Since in practice $1 < \lambda < 2.5$, it is known that $\beta < R_B/R_H < 6.25\beta$. For more precise knowledge of R_B, measurement (2) must be interpreted using the following theoretical expression, derivable by means of the theory of images:

$$\frac{R_P}{R_H} = \left(\frac{\lambda z}{2e}\right) W\left(\alpha^2, \frac{\lambda z}{2e} - \frac{\lambda - 1}{2}\right), \qquad (34)$$

where

$$W(x, y) = (1 - x)\int_0^\infty \frac{u e^{-yu} du}{1 - x e^{-u}},$$

z = the lateral spacing AO.

and

$$\alpha = \frac{R_B - R_S}{R_B + R_S} = \frac{\dfrac{R_B}{R_H} - \lambda}{\dfrac{R_B}{R_H} + \lambda}.$$

Using (33) and the definition of α to eliminate λ and α, in (34), it is possible to find an expression for R_P/R_H which is a function of z/e, β, and R_B/R_H. Such a relation may be used to determine R_B/R_H, since the other quantities are known from the four measurements above.

The nature of the above solution is shown in the plots of R_B/R_H vs R_P/R_H in Figure 10a for $z/e=2$ and in Figure 10b for $z/e=5$. Curves are plotted for several values of β (heavy curves). Also shown are curves of constant λ (thin curves).

Inspection of these curves shows that, particularly for the thin-bed case ($z/e=5$) for a given value of β, a small error in R_P/R_H can cause a large error in R_B/R_H. This means that for thin beds the method does not have much resolution for the determination of R_B. For bed thicknesses approaching the lateral spacing the resolution is somewhat better (compare case of $z/e=2$).

The above discussion cannot, of course, be taken to apply unmodified to the practical case where there is a borehole. It does point up the conclusion that there is no highly satisfactory way of correcting the lateral reading by itself to give the true resistivity of a *thin* isotropic bed between anisotropic adjacent formations.

CONCLUSION

A method of analysis has been described whereby a potential problem involving anisotropic media can be transformed into an equivalent problem involving isotropic media only. The method is subject to the limitation of equation (6) which requires that the coefficient of anisotropy λ be expressible as a product of a function of the radial co-ordinate only by a function of the axial co-ordinate only.

The potentials found by the solution of the equivalent isotropic problem, transformed back into the coordinates of the original problem, are sufficient to determine the apparent resistivities which would be observed by electrode-type logging devices in anisotropic formations.

By means of the above transformations, one can predict, qualitatively, that a short-spacing measuring device in a borehole filled with isotropic mud will read higher than for an isotropic formation of resistivity R_H (the horizontal anisotropic resistivity). This is confirmed in the case of the normal device by the accurately computed departure curves of Figure 3.

In *the case where the borehole is neglected* some conclusions related to log interpretation have been noted.

1. Both normal and lateral devices measure R_H when the sonde is completely within the bed, and not too close to a bed boundary.

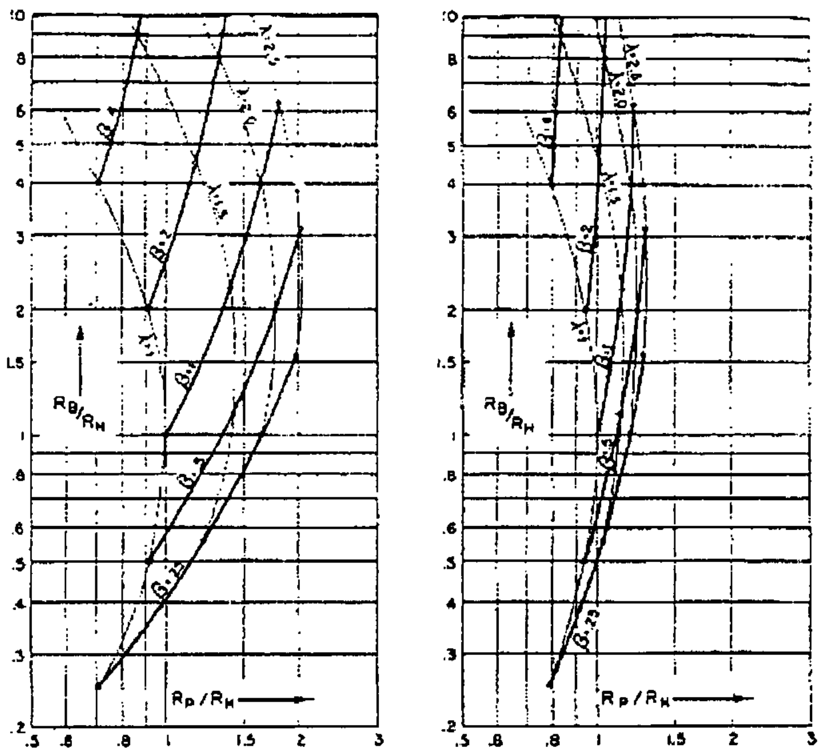


FIG. 10. Lateral sonde, borehole effect neglected, relation between R_B/R_H and R_P/R_H as a function of β (heavy curves) and λ (light curves).

2. Some anomalies occur when the electrode system is crossing a boundary between beds. In general these anomalies tend to be more striking for the lateral device.

3. The apparent resistivity shown by the lateral device shows a discontinuity when the measure electrodes cross the bed boundary. The apparent resistivities just above and just below the boundary are in proportion to the *vertical resistivities* at these points (equations (32) and (33)).

4. It is thus possible in some cases, for the sense of the discontinuity in the

lateral curve to be in the opposite direction to the change in horizontal resistivity. As a result, a direct interpretation of the log may sometimes be misleading, particularly in the case of a thin isotropic bed between thick anisotropic shoulder formations. An attempt to interpret such a case from the readings of the lateral alone indicated poor resolution in the determination of R_B, particularly for smaller bed thicknesses.

In case there is a borehole, the above conclusions can apply approximately if the sonde spacing is large in comparison with the borehole diameter.

ACKNOWLEDGMENT

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List of Symbols

- R_H Formation resistivity measured by passing current parallel to bedding planes (horizontal or longitudinal resistivity).
- R_V Formation resistivity measured by passing current in a direction perpendicular to bedding planes (vertical or transversal resistivity).
- R Mean resistivity ($R = \sqrt{R_H \cdot R_V}$).
- λ Coefficient of anisotropy ($\lambda = \sqrt{R_V / R_H}$).
- ρ Radial coordinate of a cylindrical system of coordinates.
- z Axial coordinate of a cylindrical system of coordinates. In the discussion, z is set equal to the spacing of a normal or lateral resistivity logging device.
- ρ' Radial coordinate in equivalent isotropic problem.
- z' Axial coordinate in equivalent isotropic problem.
- R' The formation resistivity in the equivalent isotropic problem.
- R_m The resistivity of the isotropic mud column filling the borehole.
- R_a The apparent resistivity indicated by a given resistivity logging device.
- R_a' The apparent resistivity which would be seen by the equivalent resistivity logging device in the equivalent isotropic problem.
- $\overline{R}$ A reference resistivity for the two-media case, ($\overline{R} = 2R_1R_2/(R_1+R_2)$), where R_1 is the mean resistivity of the upper bed and R_2 is the mean resistivity of the lower bed.
- R_B Resistivity of an isotropic bed.
- R_S Mean resistivity of anisotropic adjacent formations on either side of isotropic bed.
- R_{aB} Apparent resistivity observed in a bed just within bed boundary.
- R_{aS} Apparent resistivity observed in anisotropic adjacent formation just outside bed boundary.
- β Equal to R_{aB}/R_{aS} at either bed boundary in the no-borehole case.

REFERENCES

Doll, H. G., 1949, Introduction to induction logging and application to logging of wells drilled with oil base mud: Jour. of Petroleum Technology, v. 1, p. 148.

Filon, L. N. G., 1928–1929, On a quadrature formula for trigonometric integrals: Proc. of the Royal Society of Edinburgh, v. 49, p. 38.

Maillet, R., and Doll, H. G., 1932, Sur un théorème relatif aux milieux électriquement anisotropes et ses applications à la prospection électrique en courant continu: Ergänzungshefte für angewandte Geophysik, v. 3, p. 109.

Schlumberger, C., 1920, Etude sur la prospection électrique du sous-sol: Paris, Gauthier-Villars, p. 40.